Similarities between Timing Constraints

Towards Interchangeable Constraint Models for Real-World Software Systems

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Software for real-world systems

- **System Complexity:** guarantees of exact system behavior are impractically expensive [Lee, 2005].

- **Operating Environment:** The unpredictable nature of the environments in which software systems operate determines that their interactions with the outer world may not be totally expected [Jackson et al., 2007].

- **Computational Intractability:** From a theoretical point of view, achieving exactness in the verification of system properties is sometimes intractable [Alur and Dill, 1994].

\[ \Box (p \rightarrow \Diamond_{5} q) \]
Similarities between timed state sequences

- A **timed state sequence** is a linear structure 

\[(\delta_0, I_0), (\delta_1, I_1), (\delta_2, I_2), \ldots \text{ where } \delta_i \subseteq Prop\]

\[
\begin{array}{c|c|c|c|c|c|c|c}
\bar{\tau}_1 & \delta_0 & \delta_1 & \delta_2 & \delta_3 & \delta_4 & \ldots \\
0 & 1.1 & 2.3 & 3.3 & 4.4 & \ldots \\
\end{array}
\]

\[
\begin{array}{c|c|c|c|c|c|c|c}
\bar{\tau}_2 & \delta_0 & \delta_1 & \delta_2 & \delta_3 & \delta_4 & \ldots \\
0 & 1.2 & 2.2 & 3.4 & 4.2 & \ldots \\
\end{array}
\]

- Absolute displacement between two interval sequences \(\bar{I}\) and \(\bar{I}'\)

\[
D_a^{\bar{I}} (\bar{I}, \bar{I}') = \left[ d_{a_{\inf}}^{\bar{I}} (\bar{I}, \bar{I}'), d_{a_{\sup}}^{\bar{I}} (\bar{I}, \bar{I}') \right]
\]

where

\[
d_{a_{\sup}}^{\bar{I}} (\bar{I}, \bar{I}') = \sup \left\{ l(I'_i) - l(I_i) | i < n (\bar{I}) \right\}
\]

\[
d_{a_{\inf}}^{\bar{I}} (\bar{I}, \bar{I}') = \inf \left\{ l(I'_i) - l(I_i) | i < n (\bar{I}) \right\}
\]
Absolute $[x, y]$-tube function

Let $\bar{I}$ and $\bar{I}'$ be two interval sequences. There exists an absolute $[x, y]$-tube function from $\bar{I}$ to $\bar{I}'$ iff $D_{\alpha}^T(\bar{I}, \bar{I}') \subseteq [x, y]$
Linear timing constraints
\[
\begin{align*}
& t(e_1) - t(e_2) \leq 6, \\
& t(e_2) - t(e_1) \leq 6, \\
& t(e_1) - t(e_3) \leq 7, \\
& t(e_3) - t(e_1) \leq 3, \\
& t(e_2) - t(e_3) \leq 9, \\
& t(e_3) - t(e_2) \leq 14
\end{align*}
\]

Timed trace set

Difference relations between every pairs of events determine the shape of the trace polyhedron.
Timed Trace Inclusions and Intersections

(c) Inclusion

(d) Intersection
Similarities between timed trace sets
Proposed Metrics: Absolute Differences

Absolute differences

\[ D_a(C, C') = [d_{a_{\text{inf}}}(C, C'), d_{a_{\text{sup}}}(C, C')] \]

where

\[ d_{a_{\text{sup}}}(C, C') = \sup \left\{ d_{i,j}^* - d_{i,j}'^* \mid i = 1, \ldots, n, j = 1, \ldots, n, i \neq j \right\} \]

\[ d_{a_{\text{inf}}}(C, C') = \inf \left\{ d_{i,j}^* - d_{i,j}'^* \mid i = 1, \ldots, n, j = 1, \ldots, n, i \neq j \right\} \]

For example, in the previous slide, the absolute difference between the two timed trace sets is derived as

\[ d_{a_{\text{sup}}}(C, C') = \sup \{6 - 5, 6 - 7, 7 - 5, 3 - 2, 9 - 10, 9 - 5\} = 4, \]

\[ d_{a_{\text{inf}}}(C, C') = \inf \{6 - 5, 6 - 7, 7 - 5, 3 - 2, 9 - 10, 9 - 5\} = -1, \text{ and} \]

\[ D_a(C, C') = [-1, 4]. \]

\[ \text{Note that } d_{3,2}^* = 9 \text{ instead of 14} \]
Proposition: (This directly follows from the inclusion theorem)

- Systems satisfying timing constraint set $C$ will satisfy timing constraint set $C'$ when every constraint in $C'$ is incremented by $d_{asup}(C, C')$, i.e., for all $i \neq j$:
  $d_{i,j}^* + d_{asup}(C, C')$; and symmetrically,

- systems satisfying timing constraint set $C'$ will satisfy timing constraint set $C$ when every constraint in $C$ is incremented by $d_{asup}(C', C)$, i.e., for all $i \neq j$:
  $d_{i,j}^* + d_{asup}(C', C) = d_{i,j}^* + d_{ainf}(C, C')$.

Transitive relations can be bounded by:

$$D_a(C, C'') \subseteq \left[ d_{ainf}(C, C') + d_{ainf}(C', C''), d_{asup}(C, C') + d_{asup}(C', C'') \right]$$
Relative differences

\[ D_r (C, C') = [d_{r, \text{inf}} (C, C') , d_{r, \text{sup}} (C, C')] \]

where

\[ d_{r, \text{sup}} (C, C') = \sup \left\{ \frac{d_{i,j}^*}{d_{i,j}^*'} \mid i = 1, \ldots, n, j = 1, \ldots, n, i \neq j \right\} \]

\[ d_{r, \text{inf}} (C, C') = \inf \left\{ \frac{d_{i,j}^*}{d_{i,j}^*'} \mid i = 1, \ldots, n, j = 1, \ldots, n, i \neq j \right\} \]

For example, the relative difference between the two timed trace sets is

\[ d_{r, \text{sup}} (C, C') = \sup \{6/5, 6/7, 7/5, 3/2, 9/10, 9/5\} = 9/5, \]

\[ d_{r, \text{inf}} (C, C') = \inf \{6/5, 6/7, 7/5, 3/2, 9/10, 9/5\} = 6/7, \text{ and} \]

\[ D_r (C, C') = [6/7, 9/5]. \]

**Conjecture:** The proportion of the “volume” of the intersection in that of \( C \) is lower bounded by \( \frac{1}{d_{r, \text{sup}} (C, C')} \); and symmetrically, the proportion of the “volume” of the intersection in that of \( C' \) is lower bounded by \( \frac{1}{d_{r, \text{sup}} (C', C)} = d_{r, \text{inf}} (C, C') \).
References


