Design Verifiably Correct Model Patterns to Facilitate Modeling Medical Best Practice Guidelines with Statecharts (Technical Report)

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Abstract-Improving patient care safety is an ultimate objective for medical cyber-physical systems. A recent study shows that the patients' death rate can be significantly reduced by computerizing medical best practice guidelines. To facilitate the development of computerized medical best practice guidelines, statecharts are often used as a modeling tool because of their high resemblances to disease and treatment models and their capabilities to provide rapid prototyping and simulation for clinical validations. However, some implementations of statecharts, such as Yakindu statecharts, are priority-based and have synchronous execution semantics which makes it difficult to model certain functionalities that are essential in modeling medical guidelines, such as two-way communications and configurable execution orders. Rather than introducing new statechart elements or changing the statechart implementation's underline semantics, we use existing basic statechart elements to design model patterns for the commonly occurring issues. In particular, we show the design of model patterns for two-way communications and configurable execution orders and formally prove the correctness of these model patterns. We further use a simplified airway laser surgery scenario as a case study to demonstrate how the developed model patterns address the two-way communication and configurable execution order issues and their impact on validation and verification of medical safety properties.

Index Terms—Verifiably correct model patterns, medical guideline modeling, Statechart models.

I. INTRODUCTION AND RELATED WORK

A study shows that the patients' death rate can be significantly reduced by computerizing medical best practice guidelines [1]. Developing computerized disease and treatment models from medical best practice handbooks needs close interactions with medical professionals. In addition, to satisfy the safety and correctness requirements, the derived models also need to be clinically validated and formally verified. Over past two decades, many computer executable medical best practice guideline models are developed, such as Asbru [2], GLIF [3], GLARE [4], EON [5], and PROforma [6]. Rahmaniheris *et al.* [7] have developed an organ-centric approach to model medical best practice guidelines with state-charts, which enable rapid prototyping and allow quick clinical validations by medical staff through simulations. Tan *et al.* [8]

proposed a design pattern for wireless medical Cyber-Physical Systems (CPS). Jiang et al. [9] presented a timed automata and synchronous dataflow based framework for system design. Guo et al. [10], [11] proposed approaches to model and integrate medical resource availability and relationships in existing medical guideline models. To help improve clinical validation, Wu et al. have developed a workflow adaptation protocol [12] to help physicians safely adapt workflows to react to patient adverse events and a treatment validation protocol [13] to enforce the correct execution sequence of performing a treatment based on preconditions validation, side effects monitoring, and expected responses checking. Both the workflow adaptation and treatment validation protocol are based on pathophysiological models. In addition, based on organ-specific physiology, a system that integrates medical devices into semi-autonomous clusters in a network-failsafe manner has also been developed by Kang et al. [14]. Christov et al. [15] proposed an approach to detect whether the performed medical procedures have deviated from the recommended ways to perform the medical procedures, i.e., medical best practices. To formally verify safety properties of medical guideline models, Guo et al. [16] presented an approach to transform statecharts to timed automata. Runtime verification techniques [17], [18] were also applied to improve safety of medical guideline systems at code level.

Most existing medical best practice guidelines in hospital handbooks are represented by flowcharts [19] which are very similar to statecharts [20], so are many medical disease models and treatment models. In addition to the high similarities between medical models and statecharts, statecharts are executable and have become a widely used model in designing complex systems, such as avionics [21], air traffic control systems [22], and medical systems [7], [23], [16]. These distinguishing features of statecharts have inspired us to use it as a computerized representation for medical best practice guidelines. However, there are functionalities that are essential in medical operations, such as two-way communications and configurable execution orders, which are not directly supported by some open source statechart modeling tools, such as

Yakindu. We use the following two examples to illustrate the need of two-way communications and configurable execution orders in medical domain. By two-way communications we mean two statecharts can communicate with each other, and by configurable execution orders we mean the execution orders can be configured by users without change the model itself.

Example 1. Laser surgery [24] is a surgical procedure that uses a laser to remove problematic tissues and is widely used in airway surgery, thoracic surgery, eye surgery, etc. For airway laser surgery, there are two potential dangers: (1) an accidental burn if both laser and ventilator are activated; and (2) a low-oxygen shock if the Saturation of Peripheral Oxygen (Sp0) level of the patient decreases below a given threshold (assume 95%) [25]. To prevent the potential dangers, the airway laser and the ventilator must be able to communicate with each other bidirectionally, i.e., the communication needs to be two-way. In particular, when the surgery starts, the airway laser turns on and notifies the ventilator to turn off; and when the patient's Sp0 level becomes below 95%, the ventilator turns on and notifies the airway laser to turn off.

Example 2. In the chronic cough treatment guideline [26], chest radiography has to be performed before sputum test and bronchoscopy, but the guideline does not specify the order between sputum test and bronchoscopy. The physicians can choose the medical procedure order chest radiography \prec sputum test \prec bronchoscopy or chest radiography \prec bronchoscopy \prec sputum test based on their experiences/preferences and medical resource availability.

Without the two-way communication support, the laser and the ventilator in the airway laser surgery can be both activated and cause surgery fire. We present more details about the scenario in the case study, i.e., Section V. Without the configurable execution order support, if a physician deviate from the medical procedure execution order specified in a medical guideline system, the system can not identify whether or not the deviation is safe. Hence, supporting two-way communications and configurable execution orders are very important for modeling medical best practice guidelines.

However, most existing medical guideline modeling languages, such as Asbru [2], GLIF [3], and PROforma [27], do not support these essential functionalities. Althouh Simulink Stateflow [28], a statechart variant from Matlab [29], supports two-way communications through *condition actions* and preemptive execution semantics, it does not support execution order change without modifying existing models.

Open source Yakindu statecharts are priority-based and have synchronous execution semantics. With such execution semantics, only higher priority statecharts can send *events* to lower priority statecharts, but not the other way around. In addition, each statechart is pre-assigned an unique priority to determine its execution order when a model is established.

A naive approach to implement two-way communication functionality is to use global variables shared among multiple statecharts. However, using global variables has known disadvantages of increased difficulty and complexity to maintain data consistence. An alternative is to take a similar approach as in Stateflow [28] by introducing new elements into Yakindu to interrupt the execution of the *event* sender statechart and resume after handling the *event*. However, such approach changes Yakindu statecharts' underline execution semantics and violates Yakindu statecharts' original design goal.

To address the configurable execution order issue, a straight forward approach is to customize medical guideline statechart models based on execution orders provided by physicians. However, the approach faces the following challenges: (1) a medical guideline model requires a variant for every physician to facilitate different execution orders, which is cumbersome for practical use; and (2) engineers need to be involved in clinical care to manually modify medical guideline models when physicians change execution orders.

The major challenge of implementing the essential functionalities in modeling medical guidelines, such as two-way communications and configurable execution orders, is that it has to be effortless for medical professionals to validate its correctness and formally verifiable at reasonable cost. The paper presents an approach to apply model patterns to support these essential functionalities. The model patterns do not introduce new statechart elements nor change statecharts' underline execution semantics. Hence, the model patterns do not require additional effort for medical professionals to validate their correctness. In addition, our previous work [16] can also be applied to formally verify medical guideline models that apply the model patterns.

The rest of the paper is organized as follows. Section II briefly introduce the preliminary work on developing verifiably safe medical best practice guidelines with statecharts. We design the two-way communication model pattern and configurable execution order model pattern in Section III and formally prove the model patterns' correctness in Section IV. A case study of a simplified airway laser surgery scenario is performed in Section V. We present some discussions in Section VI and conclude in Section VII.

II. PRELIMINARY WORK

Our previous work [16] presents an approach to build verifiably safe executable medical guideline models in two steps: (1) use statecharts [20] to model medical guidelines and interact with medical professionals to validate the correctness of the medical guideline models; and (2) automatically transform medical guideline statecharts to timed automata [30] by the developed Y2U tool [16] to formally verify safety properties. In this section, we use the simplified airway laser surgery scenario in Example 1 as an example to briefly summarize the process of building verifiably safe executable medical guideline models.

A. Model Medical Guidelines with Statecharts

We use Yakindu statecharts to model the simplified airway laser surgery, as shown in Fig. 1. Both laser and ventilator are modeled as a statechart with three states (On, Off, and Syn) to represent the devices' operation status. To prevent the accidental burn danger caused by laser and ventilator are both activated, we add the Syn state to ensure that the laser/ventilator's activation procedure is delayed one step after the ventilator/laser's deactivation procedure. The initial state of the airway laser surgery system is: the laser is off and the ventilator is on to supply oxygen to the patient. When the ventilator is turned off, we assume that the patient's SpO level decreases by 1 every second.

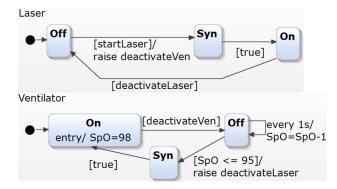


Fig. 1. Simplified Airway Laser Surgery Statechart Model

In the statechart shown in Fig. 1, when an operating surgeon sends the startLaser *event* to the laser to operate the surgery, the laser sends the deactivateVen *event* to stop oxygen supply. If the patient's SpO level reduces below 95%, the ventilator turns on to supply oxygen and sends the deactivateLaser *event* to stop the laser.

B. Verifiably Safe Statecharts

Statecharts contain basic elements, such as *states* and *transitions*, and advanced elements, such as *composite states*. Although these advanced elements provide modeling convenience, they increase the difficulty of both clinical validation and formal verification. As stated in the literature, one of the keys to achieving system safety at reasonable cost is a serious and sustained commitment to simplicity [31], [32]. To reduce the difficulty in both clinical validation and formal verification, our previous work [33] proposes a pattern-based statechart modeling approach to model medical guidelines with basic statechart elements and model patterns which are built upon these basic elements to implement advanced statechart elements.

To formally verify medical guideline statechart models, our previous work [16] presents an approach to transform statechart models to timed automata. There are three key differences between Yakindu statecharts and UPPAAL timed automata: (1) syntactic difference: Yakindu statecharts have some elements that are not directly supported by UPPAAL timed automata, such as *event* and *timing trigger*; (2) execution semantics difference: Yakindu model is deterministic and has synchronous execution semantics while the execution of UPPAAL model is non-deterministic and asynchronous; and (3) simultaneous *events* difference: Yakindu supports simultaneous events while UPPAAL does not.

Our transformation handles above three key differences by following approaches: (1) syntactic difference: define 5 transformation rules for basic Yakindu statecharts elements, i.e., *state*, *transition*, *state action*, *event*, and *timing trigger*; (2) execution semantics difference: design 2 transformation rules to implement transition trigger determinism and statechart execution determinism, and use the lockstep method [34] to force synchronous execution; and (3) simultaneous *events* difference: design an event stack to simulate simultaneous *event* mechanism in UPPAAL timed automata.

To ensure that the formal verification results in timed automata is consistent with the statecharts, we prove the transformation correctness. The approach also provides the capability to trace back paths that fail safety properties from timed automata to statecharts. Although our approach only transforms basic Yakindu statechart elements, it is sufficient to provide formal verification functionality for medical guideline statecharts because the advanced statechart elements can be represented by basic elements [33].

III. MODEL PATTERN DESIGN

Some statecharts have priority-based, deterministic, and synchronous execution semantics, such as Yakindu statecharts. The execution semantics make it difficult to model certain features that are essential in modeling medical guidelines, such as two-way communications and configurable execution orders. In this section, we design model patterns to support two-way communications and configurable execution orders without changing statecharts' underline execution semantics nor introducing new statechart elements.

A. Design Model Pattern for Two-Way Communication

In statechart models with multiple statecharts, two-way communications are essential. For instance, the laser statechart and ventilator statechart in Fig. 1 require two-way communications. We represent the two-way communication requirement by the statechart model shown in Fig. 2. The two-way communication feature means that statechart S1 can receive *event* EB raised by S2 and statechart S2 can receive *event* EA raised by S1. However, if statechart S1 has higher priority than statechart S2, the *event* EB can not be passed from S2 to S1 due to priority-based and synchronous execution semantics.

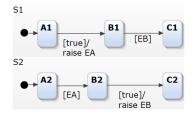


Fig. 2. Two-Way Communication Feature

In this section, we design a model pattern to support twoway communication in statecharts. The model pattern design needs to satisfy the following two criteria: (1) it does not changing statecharts' underline execution semantics; and (2) it does not introduce new statechart elements. The reason of these two criteria is that new statechart elements and new execution semantic rules increase the difficulty for medical professionals to validate medical guideline models. Based on the model pattern design criteria, our strategy to support twoway communication is to queue all raised *events*, add *logic* *execution cycles*, and re-raise the queued *events* in the added *logic cycles*. Note that each added *logic execution cycle* takes one CPU time unit which is negligible compared to clock time. Under the two-way communication model pattern, the statechart S2 in Fig. 2 can send *event* EB to statechart S1.

We take Yakindu statecharts as an example to implement the two-way communication model pattern. In particular, for each execution cycle (we call it *normal cycle*), we queue all raised events. After each normal execution cycle, we add n-1 logic cycles to re-raise queued events, where n is the number of statecharts. In each logic cycle, each queued event is only visible to statecharts whose priorities are higher than the event raiser, as lower priority statecharts have already received the queued event in the *normal cycle*. Furthermore, at each logic cycle, only those transitions that are triggered by queued events are executed. This constraint is to ensure that the logic cycle does not change the model behavior other than facilitate higher priority statecharts to receive events from lower priority statecharts.

The two-way communication model pattern contains a Manager statechart and a interface TWC, as shown in Fig. 3. In particular, the Manager statechart has the highest priority in the model and initializes the event queues. The interface TWC declares four functions: initEventQueue(), push(), pop(), and isNormalExe().

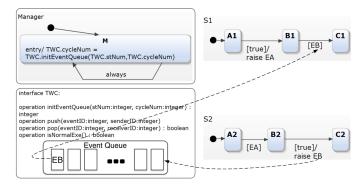


Fig. 3. Two-Way Communication Model Pattern

We define a Java class TWC shown in Fig. 4 to implement the TWC interface. The event queue is implemented by two arrays queuedEvents and queuedEventsSender to store raised events and their corresponding senders. The TWC class also includes a boolean variable normalExe that indicates if the current execution cycle is a *normal cycle* or an *logic cycle* and an integer variable queuedEventNum indicating the size of the event queue. The four functions declared in the TWC interface are implemented as follows:

- long initEventQueue(long stNum, long cycleNum) assigns the value of normalExe based on current execution cycle number cycleNum, increases the execution cycle number by 1, and clears the event queue when the execution enters *normal cycles*.
- 2) void push(long event, long sender) pushes a raised event and the event raiser into the event queue.
- boolean pop(long event, long receiver) checks if the input event is valid. In *normal cycles*, events raised by higher priority stetecharts are valid; while in *logic*

cycles, events raised by lower priority stetecharts are valid.

 boolean isNormalExe() checks if the current execution cycle is a normal cycle.

The TWC class only uses basic Java data types, such as integer, boolean, and integer array, and basic Java statements, such as assignment, *if* statement, and *while* statement. The implementation principle of only using basic elements decreases the difficulty of correctness proof in Section IV-A.

```
private long[] queuedEvents;
private long[] queuedEventsSender;
private int queuedEventNum = 0;
private boolean normalExe = false;
public long initEventQueue(long stNum, long cycleNum){
    long a,x;
    if(cycleNum == 0) {normalExe = true;
        queuedEventNum = 0;
    else
            normalExe = false:
    a = cvcleNum:
    if(cycleNum == stNum - 1)
                                cycleNum = 0:
    else
            cycleNum = a + 1;
    x = cycleNum;
    return x;
public void push(long event, long sender){
    queuedEvents[queuedEventNum] = event;
    queuedEventsSender[queuedEventNum] = sender;
    queuedEventNum++;
public boolean pop(long event, long receiver){
    boolean x = false; int i = 0;
                                    int v = 0;
    while(i < queuedEventNum) {</pre>
        if(queuedEvents[i] == event &&
            ((normalExe&&receiver>queuedEventsSender[i])||
            (!normalExe&&receiver<queuedEventsSender[i])))
            v = i; x = true; }
        i++:
    }
    return x;
public boolean isNormalExe(){
    boolean x = normalExe;
    return x:
}
```

Fig. 4. Two-Way Communication Interface Implementation

We define Procedure 1 to apply the two-way communication model pattern in existing statechart models.

Procedure 1.

- Step 1: add the interface TWC and the Manager statechart;
- Step 2: replace each event raise action by TWC.push(TWC.eventID, TWC.senderID);
- Step 3: modify each transition's guard G as follows:
 - if G does not contain any event, replace G with G && TWC.isNormalExe();
 - if G contains events, replace each event part in G with corresponding expression TWC.pop(TWC.eventID, TWC.receiverID).

B. Design Model Pattern for Configurable Execution Order

In statechart models with multiple statecharts, end users may want to configure statechart execution orders based on their experiences and preferences. We represent configurable execution order requirement by the statechart model shown in Fig. 5. If the execution order is $S1 \prec S2$, the action x = x + 1 is executed first. While if the execution order is $S2 \prec S1$, the action y = y + 1 is executed first. However,

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the statechart execution orders (represented by priorities) are pre-assigned when building statechart models and can not be changed without modifying existing statecharts.

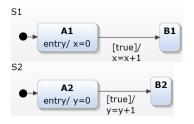


Fig. 5. Configurable Execution Order Feature

In this section, we design a model pattern to allow end users to configure statechart execution orders without modifying existing statechart models. The design criteria is the same with the two-way communication model pattern presented in Section III-A. Based on the model pattern design criteria, our strategy to support configurable execution order is to represent user specified execution order in configure files, add *logic execution cycles*, and apply token-based ordering to achieve desired execution orders.

We also take Yakindu statecharts as an example to implement the configurable execution order model pattern. In particular, the model pattern generates a token for each Yakindu execution cycle based on a specified order. For each *normal execution cycle*, we add n - 1 *logic cycles*, where n is the number of statecharts. During an execution cycle, only the statechart whose priority matches the generated token will execute one step.

Similar to the two-way communication model pattern in Section III-A, the configurable execution order model pattern contains a Manager statechart and a interface CEO, as shown in Fig. 6. In particular, the Manager statechart has the highest priority in the model and updates the execution token in each execution cycle. The interface CEO declares two functions: updateExeInfo() and run().

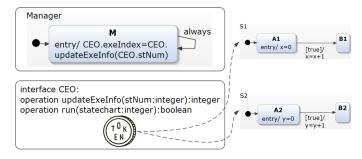


Fig. 6. Configurable Execution Order Model Pattern

We define a Java class CEO shown in Fig. 7 to implement the CEO interface. The CEO class includes an integer variable exeIndex to represent the execution token and an array exeOrders to store user specified execution orders, respectively. The two functions declared in the CEO interface are implemented as follows:

 long updateExeInfo(long stNum) updates the execution token, if the token is equal to the statechart number stNum, the token is reset to be 1, otherwise the token is increased by 1;

 boolean run(long st) checks if the input statechart st matches the current execution token.

The CEO class also only uses basic Java data types and basic Java statements, which decreases the difficulty of correctness proof in Section IV-B.

```
private long[] exeOrders;
private long exeIndex = 0;
public long updateExeInfo(long stNum){
    long x, a:
    if(exeIndex == stNum)
                             a = 1:
    else
            a = exeIndex + 1;
    x = a;
    return x:
1
public boolean run(long st){
    boolean x;
    if(exeOrders[(int) (exeIndex-1)] == st)
        x = true;
    else x = false;
    return x;
}
```

Fig. 7. Configurable Execution Order Interface Implementation

We define Procedure 2 to apply the configurable execution order model pattern in existing statechart models.

Procedure 2.

- Step 1: add the interface CEO and the Manager statechart;
- Step 2: replace each transition's guard G by G && CEO.run(CEO.statechartID).

In summary, we design two model patterns to support two-way communication and configurable execution order in statecharts. Both model patterns are implemented with basic statechart elements and external Java codes. The approach has three advantages: (1) it does not change statechart syntax and its underline execution semantics, hence the two model patterns can be applied to all statechart models; (2) it does not increase the difficulty of clinical validation and formal verification; (3) existing work on modeling, validating, and verifying medical guidelines with statecharts can be applied. For safety-critical medical systems, the correctness of the designed model patterns is crucial. We formally prove the two model patterns' correctness in Section IV.

IV. MODEL PATTERN CORRECTNESS PROOF

The model patterns designed in Section III are implemented with basic statechart elements and external Java code. Our previous work [16] have proved that the transformation of basic statechart elements to timed automata is correct. In this section, we prove the correctness of designed model patterns in two steps: (1) prove that the Java implementation is correct; and (2) transform model patterns to timed autoamta to formally verify that desired properties hold. The strategy to prove the Java implementation is as follows: (1) represent each Java function by a WHILE program [35]; (2) construct a Hoare triple [35] for each WHILE program; and (3) prove the correctness of the Hoare triples.

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A. Two-Way Communication Model Pattern

We represent the Java functions initEventQueue(), push(), pop(), and isNormalExe() by WHILE programs Program 1, Program 2, Program 3, and Program 4, respectively. The variables used to implement the two-way communication model pattern are listed in Table I. We construct Hoare triples and prove their correctness in Lemma 1-4. The precondition and postcondition of each Hoare triple are the input and output of the corresponding WHILE program based on the functionality.

Program 1.

$$\begin{aligned} \texttt{initEventQueue} &\equiv \\ & \textit{if } c = 0 \textit{ then } \texttt{exe} := \texttt{true}; \textit{ } n := 0; \\ & \textit{else } \texttt{exe} := \texttt{false}; \textit{ fi}; \\ & a := c; \\ & \textit{if } c = \texttt{stNum} - 1 \textit{ then } c := 0; \\ & \textit{else } c := a + 1; \textit{ fi}; \\ & x := c \end{aligned}$$

The input of the initEventQueue program is the statechart number and cycle number. The two-way communication requires as least two statecharts, hence stNum > 1. As the two-way communication model pattern adds stNum - 1 logic cycle and treats the normal cycle as 0, hence c < stNum. Therefore, the precondition of the initEventQueue program is $c < \texttt{stNum} \land \texttt{stNum} > 1$. According to the functionality, the initEventQueue program has two possible outputs: (1) if the current execution cycle is a *normal cycle*, then program assigns the number of next cycle to be 1 and clears the event queue, i.e., $x = 1 \land exe = true \land n = 0$; and (2) if the current execution cycle is a logic cycle, then program increases the number of next cycle by 1; if the current execution cycle is the last logic cycle, then program assigns the number of next cycle to be 0, i.e., $(x = 0 \lor x = a + 1) \land exe = false$. Hence, the the postcondition of the initEventQueue program is (x = $1 \wedge exe = true \wedge n = 0) \lor ((x = 0 \lor x = a+1) \wedge exe = false).$ We prove the correctness of the initEventQueue program in Lemma 1.

Lemma 1. { $c < stNum \land stNum > 1$ }initEventQueue{($x = 1 \land exe = true \land n = 0$) \lor (($x = 0 \lor x = a + 1$) $\land exe = false$)}

Proof. According to the composition rule [35], the proof of the lemma is equivalent to prove the following two triples:

$$\{P\}S_1; a := c\{R\}$$
(1)

$$\{R\}S_2; x := c\{Q\}$$
(2)

where $P \equiv c < \text{stNum} \land \text{stNum} > 1$, $Q \equiv (x = 1 \land \text{exe} = \text{true} \land n = 0) \lor ((x = 0 \lor x = c + 1) \land \text{exe} = \text{false})$, $R \equiv (a = c \land a = 0 \land \text{exe} = \text{true} \land n = 0 \land \text{stNum} > 1) \lor (a = c \land a < \text{stNum} \land \text{exe} = \text{false} \land \text{stNum} > 1)$,

$$S_1 \equiv$$
 if $c = 0$ then exe := true; $n := 0$;
else exe := false; fi

and

$$S_2 \equiv$$
 if $c = \texttt{stNum} - 1$ then $c := 0;$
else $c := a + 1;$ fi

The detailed proof of triple (1) and triple (2) is given in Appendix A-A. \Box

Program 2.

$$\texttt{push} \equiv E[n] := e; \ S[n] := s; \ n := n+1$$

The input of the push program is an event e, its sender s, the event array E[], and the sender array S[]. Suppose the number of raised *events* before pushing event e is N, then $n = N \land N \ge 0$. The event e and its sender s are represented with positive integers, hence $e > 0 \land s > 0$. The precondition of the push program is $n = N \land N \ge 0 \land e > 0 \land s > 0$. The push program pushes an event and its sender into the queues and increase the number of raised *events* by 1. The event array E[] and the sender array S[] have the same size n, i.e., the number of raised *events*. As the index number of arrays implementing queues starts from 0, the postcondition of the push program is $E[n-1] = e \land S[n-1] = s \land n = N+1$. We prove the correctness of the push program in Lemma 2.

Lemma 2. { $n = N \land N \ge 0 \land e > 0 \land s > 0$ }push{ $E[n-1] = e \land S[n-1] = s \land n = N+1$ }

Proof.

$$E[n-1] = e \land S[n-1] = s \land$$

$$n = N + 1[n := n+1][S[n] := s][E[n] := e]$$

$$\equiv E[n] = e \land S[n] = s \land n = N[S[n] := s][E[n] := e]$$

$$\equiv E[n] = e \land n = N[E[n] := e]$$

$$\equiv n = N$$

As $n = N \land N \ge 0 \land e > 0 \land s > 0 \rightarrow n = N$, hence the lemma is correct.

Program 3.

$$\begin{array}{ll} \texttt{pop} \equiv & x := \texttt{false}; \; i := 0; \; v := 0; \\ & \texttt{while}\; i < n \; do \\ & \texttt{if}\; E[i] = e \land ((\texttt{exe} = \texttt{true} \land r > S[i]) \lor \\ & (\texttt{exe} = \texttt{false} \land r < S[i])) \; \texttt{then} \\ & v := i; \; x := \texttt{true}; \; \texttt{fi} \; ; \\ & i := i + 1; \\ & \texttt{od} \end{array}$$

The input of the pop program is an event and its receiver, hence, the precondition of the pop program is $e > 0 \land r > 0$. The pop program checks if an event is valid to a receiver during current execution cycle. In a *normal cycle*, events raised by higher priority stetecharts are valid, i.e., $x = \texttt{true} \land E[v] = e \land 0 \le i \le n \land \texttt{exe} = \texttt{true} \land r > S[v]$, where v is event e's index in event array E. Note that a *event* e and its sender s have the same index in the event array E[] and the sender array S[], respectively. While in an *logic cycle*, events raised by lower priority stetecharts are valid, i.e., $x = \texttt{true} \land E[v] = e \land 0 \le i \le n \land \texttt{exe} = \texttt{false} \land r < S[v]$. Note that the

 TABLE I

 VARIABLES IN TWO-WAY COMMUNICATION MODEL PATTERN IMPLEMENTATION

Variable in Yakindu Java Code	Variable in WHILE Program	Variable Type	Variable Meaning		
queuedEvents[]	E[]	int array	raised events		
queuedEventsSender[]	S[]	int array	the event senders of corresponding element in		
			queuedEvents[]		
queuedEventNum	n	int	number of raised event, i.e., size of $E[]$ and $S[]$		
normalExe	exe	bool	a variable indicates if the current execution cycle is a normal		
			cycle or an additional cycle		
stNum	stNum	int	statechart number of the given model		
cycleNum	c	int	the number of execution cycle		
event	e	int	event identity		
sender	8	int	event sender		
receiver	r	int	event receiver		
v	v	int	event e's index in $E[]$ and $S[]$		
i	i	int	iterator of integer arrays $E[]$ and $S[]$		
a	a	int	temporary variable		
x	x	int/bool	the return of functions		

higher priority a statechart has, the lower its identity is. If the input event is not raised, the pop program returns false, i.e., x = false. Therefore, the postcondition of the pop program is $x = \texttt{false} \lor (x = \texttt{true} \land E[v] = e \land ((\texttt{exe} = \texttt{true} \land r > S[v]) \lor (\texttt{exe} = \texttt{false} \land r < S[v]))$. We prove the correctness of the pop program in Lemma 3.

Lemma 3. $\{e > 0 \land r > 0\}$ pop $\{x = \texttt{false} \lor (x = \texttt{true} \land E[v] = e \land ((\texttt{exe} = \texttt{true} \land r > S[v]) \lor (\texttt{exe} = \texttt{false} \land r < S[v])))\}$

Proof. The loop invariant and bound function of the while loop in the pop program are $\mathbf{inv} \equiv 0 \le i \le n \land (x = \mathtt{false} \lor (x = \mathtt{true} \land E[v] = e \land ((\mathtt{exe} = \mathtt{true} \land r > S[v]) \lor (\mathtt{exe} = \mathtt{false} \land r < S[v])))$ and $\mathbf{bd} \equiv n - i$, respectively.

We use P and Q to denote the precondition and postcondition of the triple, i.e., $P \equiv e > 0 \land r > 0$ and $Q \equiv x = \texttt{false} \lor (x = \texttt{true} \land E[v] = e \land ((\texttt{exe} = \texttt{true} \land r > S[v]) \lor (\texttt{exe} = \texttt{false} \land r < S[v])))$. We prove the lemma in the following five steps.

Step 1: prove that the initialization establishes the loop invariant, i.e., $\{P\}x := \texttt{false}; i := 0; v := 0\{\texttt{inv}\}.$

Step 2: prove that the loop body does not change the loop invariant, i.e., $\{inv \land i < n\}S\{inv\}$, where

$$\begin{split} S &\equiv & \text{if } E[i] = e \land (\texttt{exe} = \texttt{true} \land r > S[i]) \lor \\ & (\texttt{exe} = \texttt{false} \land r < S[i]) \texttt{ then} \\ & v := i; \; x := \texttt{true}; \; \texttt{fi}; \\ & i := i+1 \end{split}$$

Step 3: prove that the bound function decreases after each iteration, i.e., $\{inv \land i < n \land bd = z\}S\{bd < z\}$, where z is an integer.

Step 4: prove that the loop invariant implies the bound function is non-negative, i.e., $inv \rightarrow bd \ge 0$.

Step 5: prove that the postcondition holds when the while loop terminates, i.e., $inv \land \neg(i < n) \rightarrow Q$.

The detailed proof of each step is given in Appendix A-B. \Box

Program 4.

 $\texttt{isNormalExe} \equiv x := \texttt{exe}$

The isNormalExe program does not have input, hence, the precondition is true. As the isNormalExe program checks if the current execution cycle is a *normal cycle*, the postcondition is x = exe. We prove the correctness of the isNormalExe program in Lemma 4.

Lemma 4. {true}isNormalExe{x = exe}

Proof.

$$\begin{aligned} x &= \exp[x := \exp] \\ &\equiv \exp = \exp \\ &\equiv \texttt{true} \end{aligned}$$

After proving that the Java implementation is correct, we prove the correctness of the two-way communication model pattern in Theorem 1.

Theorem 1. The two-way communication model pattern is correct, i.e., two statecharts can send events to each other.

Proof. For the statechart model shown in Fig. 2, the communication from statechart S1 to S2 means that statechart S2 can receive *event* EA sent by statechart S1 and reach state B2. Similarly, the communication from statechart S2 to S1 is that statechart S1 can receive *event* EB sent by statechart S2 and reach state C1. Hence, to prove the correctness of the two-way communication model pattern is equivalent to prove that both state C1 and state B2 are reachable.

We have prove the correctness of transformation of statechart elements in [16] and correctness of the Java implementation in Lemma 1-4. Hence, we can transform the statechart model in Fig. 2 with two-way communication model pattern to timed automata to verify the properties. The two state reachability properties can be formally verified with TCTL (timed computation tree logic) [30] formulas E <> S1.C1 and E <> S2.B2, respectively. The verification results show that both properties are satisfied. Therefore, the two-way communication model pattern is correct.

B. Configurable Execution Order Model Pattern

We represent the Java functions updateExeInfo(), and run() by WHILE programs Program 5 and Program 6, respectively. The variables used to implement the configurable execution order model pattern are listed in Table II. We construct Hoare triples and prove their correctness in Lemma 5-6.

Program 5.

$$\label{eq:updateExeInfo} \begin{array}{ll} \texttt{if} \ t = \texttt{stNum} \ then \ a := 1; \\ \texttt{else} \ a := t + 1; \ \texttt{fi} \ ; \\ x := a \end{array}$$

The input of the updateExeInfo program is the statechart number and the execution token, hence, the precondition of the updateExeInfo program is $t > 0 \land stNum > 0$. The updateExeInfo program updates the execution token. If the current execution token is equal to the statechart number, then the program assigns the token to be 1, i.e., $t = stNum \land x = 1$; otherwise, the program increases the token by 1, i.e., $t \neq stNum \land x = t + 1$. Hence, the postcondition of the updateExeInfo program is $(t = stNum \land x =$ $1) \lor (t \neq stNum \land x = t + 1)$. We prove the correctness of the updateExeInfo program in Lemma 5.

Lemma 5. $\{t > 0 \land stNum > 0\}$ updateExeInfo $\{(t = stNum \land x = 1) \lor (t \neq stNum \land x = t + 1)\}$

Proof. According to the composition rule [35], the proof of the lemma is equivalent to prove the following two triples:

$$\{P\}S\{R\}\tag{3}$$

$$\{R\}x := a\{Q\} \tag{4}$$

where $P \equiv t > 0 \land \text{stNum} > 0$, $Q \equiv (t = \text{stNum} \land x = 1) \lor (t \neq \text{stNum} \land x = t + 1)$, $R \equiv (t = \text{stNum} \land a = 1) \lor (t \neq \text{stNum} \land a = t + 1)$, and

$$S \equiv$$
 if $t =$ stNum then $a := 1$;
else $a := t + 1$; fi

The detailed proof of triple (3) and triple (4) is given in Appendix A-C. $\hfill \Box$

Program 6.

$$run \equiv if O[t-1] = st then x := true;$$
$$else x := false; fi$$

The input of the run program is a statechart's identity and an execution token. As we define that the minimal statechart identity and minimal generated execution token are both 1, hence the precondition of the run program is $t > 0 \land st > 0$. The run program checks if a statechart is valid to execute, i.e., the statechart's identity matches the current execution token. The array O[] stores statechart identities sorted in their execution orders. The index number of the array O[] starts from 0. Hence, the postcondition of the run program is $(x = \texttt{true} \land O[t-1] = \texttt{st}) \lor (x = \texttt{false} \land O[t-1] \neq \texttt{st}).$ We prove the correctness of the run program in Lemma 6.

Lemma 6. $\{t > 0 \land st > 0\}$ run $\{(x = true \land O[t - 1] = st) \lor (x = false \land O[t - 1] \neq st)\}$

Proof. The proof of the lemma is equivalent to prove the following two triples:

$$\{P \land O[t-1] = \mathtt{st}\}x := \mathtt{true}\{Q\} \tag{5}$$

$$\{P \land O[t-1] \neq \mathtt{st}\}x := \mathtt{false}\{Q\} \tag{6}$$

where $P \equiv t > 0 \land \text{st} > 0$ and $Q \equiv (x = \text{true} \land O[t-1] = \text{st}) \lor (x = \text{false} \land O[t-1] \neq \text{st}).$

The detailed proof of triple (5) and triple (6) is given in Appendix A-D. $\hfill \Box$

After proving that the Java implementation is correct, we prove the correctness of the configurable execution order model pattern in Theorem 2.

Theorem 2. The configurable execution order model pattern is correct, i.e., the statechart execution order is the same with the configuration.

Proof. For the statechart model shown in Fig. 5, if the execution order is $S1 \prec S2$, the action x = x + 1 is executed before y = y+1, i.e., $x \ge y$. If the execution order is $S2 \prec S1$, the action y = y+1 is executed before x = x+1, i.e., $y \ge x$. Hence, to prove the correctness of the configurable execution order model pattern is equivalent to prove the above two properties under corresponding execution order configurations.

We have prove the correctness of transformation of statechart elements in [16] and correctness of the Java implementation in Lemma 5 and Lemma 6. Hence, we can transform the statechart model in Fig. 5 with configurable execution order model pattern to timed automata to verify the properties. The two properties can be formally verified with TCTL (timed computation tree logic) [30] formulas $A[] x \ge y$ and $A[] y \ge x$, respectively. We specify statechart execution orders to be S1 \prec S2 and S2 \prec S1. The verification results show that the properties hold under corresponding execution order configurations. Therefore, the configurable execution order model pattern is correct.

V. CASE STUDY

In this section, we use the simplified airway laser surgery scenario presented in Example 1 to demonstrate how the designed model patterns can facilitate developing executable medical guideline models and their impact on validation and verification of medical safety properties.

The simplified airway laser surgery scenario has two safety properties, i.e., **P1**: the laser and the ventilator can not be activated at the same time; and **P2**: the patient's Sp0 level can not be smaller than 95%. We run simulation of the simplified airway laser surgery statechart model in Fig. 1 through Yakindu. The simulation results show that the model reaches an unsafe state that both Laser and Ventilator are on. In addition, we use the Y2U tool [16] to automatically

 TABLE II

 VARIABLES IN CONFIGURABLE EXECUTION ORDER MODEL PATTERN IMPLEMENTATION

Variable in Yakindu Java Code	Variable in WHILE Program	Variable Type	Variable Meaning
exeOrders[]	<i>O</i> []	int array	statechart execution orders
exeIndex	t	int	execution token
stNum	stNum	int	statechart number of the given model
st	st	int	statechart identity
a	a	int	temporary variable
x	x	int/bool	the return of functions

transform the statechart model in Fig. 1 to timed automata to formally verify the two safety properties. The safety properties **P1** and **P2** are verified in UPPAAL by TCTL [30] formulas A[] !(Laser.On && Ventilator.On) and A[] Sp0 >= 95, respectively. The verification results show that the safety property **P1** indeed fails. We trace back the execution path that fails **P1** to the statechart model and find that the deactivateLaser *event* sent by ventilator can not be received by the laser. The reason is that Yakindu statecharts' priority-based execution semantics cause that lower priority statecharts can not send *events* to higher priority statecharts.

We use Procedure 1 and Procedure 2 to apply the two-way communication model pattern and the configurable execution order model pattern to the statechart model in Fig. 1. The simplified airway laser surgery statechart with model patterns is shown in Fig. 8. We run simulation of the statechart model in Fig. 8 through Yakindu. The simulation results show that the model does not reach any unsafe states. We also transform the statechart model in Fig. 8 to timed automata, as shown in Fig. 9. The verification results also indicate that both safe properties **P1** and **P2** are satisfied.

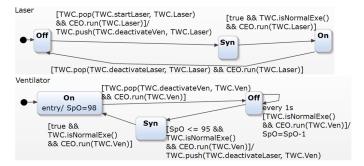


Fig. 8. Simplified Airway Laser Surgery Statecharts with Model Patterns

The default statechart execution order of the statechart model in Fig. 8 is Laser \prec Ventilator. Some medical professionals may prefer the execution order to be Ventilator \prec Laser. We specify the statechart execution order to be Ventilator \prec Laser and validate/verify the two safety properties. Both simulation and verification results show that **P1** and **P2** are still satisfied.

We also use the simplified airway laser surgery to illustrate the correctness of designed model patterns when they are accompanied by medical guideline models. The correctness of the two-way communication model pattern is that two statecharts can send events to each other. For the simplified airway laser surgery model shown in Fig. 8, the communication from statechart Laser to Ventilator means that statechart

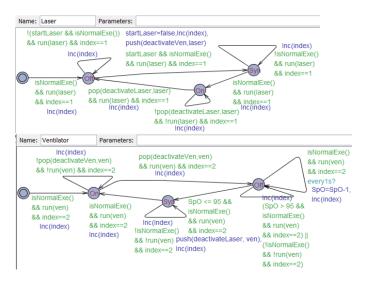


Fig. 9. Simplified Airway Laser Surgery Timed Automata Model with Model Patterns

Ventilator can receive *event* deactivateVen sent by statechart Laser and reach state Ventilator.Off. Similarly, the communication from statechart Ventilator to Laser is that statechart Laser can receive *event* deactivateLaser sent by statechart Ventilator and reach state Laser.Off. Hence, the correctness of two-way communication model pattern is that both state Ventilator.Off and state Laser.Off are reachable. We verify the two state reachability properties in the timed automata model shown in Fig. 9 with TCTL formulas $E \ll$ Ventilator.Off and $E \ll$ Laser.Off, respectively. The verification results show that both properties are satisfied, i.e., the two-way communication model pattern is correct in the simplified airway laser surgery model.

The correctness of the configurable execution order model pattern is that the statechart execution order is the same with the configuration. For the simplified airway laser surgery model shown in Fig. 8, if the execution order is Laser \prec Ventilator, the initial state Laser.Off is reached before the initial state Ventilator.On, which is represented by TCTL formula A[] Ventilator.On *imply* Laser.Off. If the execution order is Ventilator \prec Laser, the initial state Ventilator.On is reached before the initial state Ventilator.On is reached before the initial state Ventilator.On is reached before the initial state Laser.Off, which is represented by TCTL formula A[] Laser.Off *imply* Ventilator.On. We specify statechart execution orders to be Laser \prec Ventilator and Ventilator \prec Laser and verify corresponding properties in the timed automata model shown in Fig. 9, respectively. The verification results show that the properties hold under corre-

sponding execution order configurations, i.e., the configurable execution order model pattern is correct in the simplified airway laser surgery model.

The case study demonstrates that (1) the two-way communication and configurable execution order functionalities are crucial in modeling medical guidelines; and (2) the designed model patterns can address the two-way communication and configurable execution order issues.

VI. DISCUSSION

The designed model patterns support important functionalities in modeling systems with statecharts, i.e., the two-way communication functionality and the configurable execution order functionality. They can be directly applied to any application scenario that requires two-way communication or configurable execution order. If a statechart model needs to apply multiple model patterns, we can just apply these model patterns one by one. For example, the airway laser surgery statechart model shown in Fig. 8 first applies the two-way communication model pattern and then applies the configurable execution order model pattern according to Procedure 1 and Procedure 2, respectively. Given a statechart model with Ntransitions, the time complexity of applying the model patterns is O(N).

The model patterns contain two major parts: a Manager statechart and a *interface*. If we want to implement the model patterns in other statechart-based modeling platforms such as Stateflow [28], we only need to re-implement the *interface* based on corresponding modeling platforms' features. The Manager statechart does not need to be re-designed, as it only contains basic statechart elements, i.e., *state*, *transition*, *guard*, and *action*. The proposed model pattern design approach is generalized and can be applied to facilitate other functionalities that are not directly supported by statecharts, such as exception handling.

The correctness of the model patterns is proved in Theorem 1 and Theorem 2. We also use the simplified airway laser surgery case study to illustrate the correctness of the model patterns when they are accompanied by medical guideline models. To ensure the completeness of the model patterns' correctness proof, we need to verify below properties when the model patterns are applied in a statechart model. For the two-way communication model pattern, we need to verify that every *event* sent by a statechart can be received by all other statecharts in the model. Regarding the configurable execution order model pattern, for all possible statechart execution order configurations, we need to verify that every statechart's initial state is reached in sequence as configured.

VII. CONCLUSION

Some essential functionalities in medical operations, such as two-way communication and configurable execution order, are not directly supported by some open source statechart modeling tools. The paper presents an approach to apply model patterns to support these essential functionalities and formally prove the correctness of designed model patterns. The approach can be applied to facilitate other functionalities that are not directly supported by statecharts, such as exception handling. The designed model patterns can be directly applied in many application domains in which these functionalities are needed.

APPENDIX A Proof of the Lemmas

A. Proof of Lemma 1

Proof. The proof of triple (2) is as follows.

$$\begin{split} Q[x := c] \\ &\equiv (x = 1 \land \texttt{exe} = \texttt{true} \land n = 0) \lor \\ &((x = 0 \lor x = c + 1) \land \texttt{exe} = \texttt{false})[x := c] \\ &\equiv (c = 1 \land \texttt{exe} = \texttt{true} \land n = 0) \lor \\ &((c = 0 \lor c = c + 1) \land \texttt{exe} = \texttt{false}) \\ &\equiv (c = 1 \land \texttt{exe} = \texttt{true} \land n = 0) \lor \\ &(c = 0 \land \texttt{exe} = \texttt{false}) \\ &\equiv Q_1 \end{split}$$

To prove $\{R\}S_2\{Q_1\}$ is equivalent to prove the following two triples:

$$\{R \land c = \texttt{stNum} - 1\}c := 0\{Q_1\} \tag{7}$$

$$[R \wedge c \neq \mathtt{stNum} - 1]c := a + 1\{Q_1\}$$
(8)

The proof of triple (7) is as follows.

{

$$Q_1[c := 0]$$

$$\equiv (c = 1 \land exe = true \land n = 0) \lor (c = 0 \land exe = false)[c := 0]$$

$$\equiv (0 = 1 \land exe = true \land n = 0) \lor (0 = 0 \land exe = false)$$

$$\equiv exe = false$$

$$\equiv Q_2$$

$$\begin{split} R \wedge c &= \texttt{stNum} - 1 \\ &\equiv \left((a = c \wedge a = 0 \wedge \texttt{exe} = \texttt{true} \wedge n = 0 \wedge \texttt{stNum} > 1 \right) \vee \\ &(a = c \wedge a < \texttt{stNum} \wedge \texttt{exe} = \texttt{false}) \wedge \texttt{stNum} > 1) \wedge \\ &c = \texttt{stNum} - 1 \\ &\equiv \texttt{false} \vee (a = c \wedge c = \texttt{stNum} - 1 \wedge \texttt{exe} = \texttt{false} \wedge \\ &\texttt{stNum} > 1) \end{split}$$

$$\equiv a = c \wedge c = \texttt{stNum} - 1 \wedge \texttt{exe} = \texttt{false} \wedge \texttt{stNum} > 1$$

As $R \wedge c = stNum - 1 \rightarrow Q_2$, hence the triple (7) is correct. The proof of triple (8) is as follows.

$$\begin{aligned} Q_1[c := a + 1] \\ &\equiv (c = 1 \land \texttt{exe} = \texttt{true} \land n = 0) \lor \\ &(c = 0 \land \texttt{exe} = \texttt{false})[c := a + 1] \\ &\equiv (a + 1 = 1 \land \texttt{exe} = \texttt{true} \land n = 0) \lor \\ &(a + 1 = 0 \land \texttt{exe} = \texttt{false}) \\ &\equiv (a = 0 \land \texttt{exe} = \texttt{true} \land n = 0) \lor \\ &(a = -1 \land \texttt{exe} = \texttt{false}) \\ &\equiv Q_3 \end{aligned}$$

$$\begin{split} R \wedge c \neq \mathtt{stNum} - 1 \\ \equiv \left((a = c \wedge a = 0 \wedge \mathtt{exe} = \mathtt{true} \wedge n = 0 \wedge \mathtt{stNum} > 1 \right) \lor \\ (a = c \wedge a < \mathtt{stNum} \wedge \mathtt{exe} = \mathtt{false} \wedge \mathtt{stNum} > 1)) \land \\ c \neq \mathtt{stNum} - 1 \\ \equiv \left(a = c \wedge a = 0 \wedge \mathtt{exe} = \mathtt{true} \wedge n = 0 \wedge \mathtt{stNum} > 1 \right) \lor \\ (a = c \wedge a < \mathtt{stNum} - 1 \wedge \mathtt{exe} = \mathtt{false} \wedge \mathtt{stNum} > 1) \end{split}$$

As $R \wedge c \neq \texttt{stNum} - 1 \rightarrow Q_3$, hence the triple (8) is correct. Therefore, the triple (2) is correct.

Similarly, the proof of triple (1) is as follows.

$$\begin{split} R[a:=c] \\ &\equiv (a=c \wedge a=0 \wedge \texttt{exe}=\texttt{true} \wedge n=0 \wedge \texttt{stNum} > 1) \lor \\ &(a=c \wedge a < \texttt{stNum} \wedge \texttt{exe}=\texttt{false} \wedge \texttt{stNum} > 1)[a:=c] \\ &\equiv (c=0 \wedge \texttt{exe}=\texttt{true} \wedge n=0 \wedge \texttt{stNum} > 1) \lor \\ &(c < \texttt{stNum} \wedge \texttt{exe}=\texttt{false} \wedge \texttt{stNum} > 1) \\ &\equiv R_1 \end{split}$$

To prove $\{P\}S_1\{R_1\}$ is equivalent to prove the following two triples:

$$\{P \land c = 0\}$$
exe := true; $n := 0\{R_1\}$ (9)

$$\{P \land c \neq 0\} \texttt{exe} := \texttt{false}\{R_1\} \tag{10}$$

The proof of triple (9) is as follows.

$$\begin{split} R_1[n := 0][\texttt{exe} := \texttt{true}] \\ \equiv (c = 0 \land \texttt{exe} = \texttt{true} \land n = 0 \land \texttt{stNum} > 1) \lor (c < \texttt{stNum} \\ \land \texttt{exe} = \texttt{false} \land \texttt{stNum} > 1)[n := 0][\texttt{exe} := \texttt{true}] \\ \equiv (c = 0 \land \texttt{exe} = \texttt{true} \land \texttt{stNum} > 1) \lor \\ (c < \texttt{stNum} \land \texttt{exe} = \texttt{false} \land \texttt{stNum} > 1)[\texttt{exe} := \texttt{true}] \\ \equiv c = 0 \land \texttt{stNum} > 1 \\ \equiv R_2 \end{split}$$

$$\begin{split} P \wedge c &= 0 \\ &\equiv c < \texttt{stNum} \wedge \texttt{stNum} > 1 \wedge c = 0 \\ &\equiv c = 0 \wedge \texttt{stNum} > 1 \end{split}$$

As $P \wedge c = 0 \equiv R_2$, hence the triple (9) is correct. The proof of triple (10) is as follows.

$$\begin{split} &R_1[\texttt{exe}:=\texttt{false}]\\ &\equiv (c=0 \land \texttt{exe}=\texttt{true} \land n=0 \land \texttt{stNum} > 1) \lor \\ &(c<\texttt{stNum} \land \texttt{exe}=\texttt{false} \land \texttt{stNum} > 1)[\texttt{exe}:=\texttt{false}]\\ &\equiv c<\texttt{stNum} \land \texttt{stNum} > 1\\ &\equiv R_3 \end{split}$$

$$\begin{split} P \wedge c \neq 0 \\ \equiv c < \texttt{stNum} \wedge \texttt{stNum} > 1 \wedge c \neq 0 \end{split}$$

As $P \wedge c \neq 0 \rightarrow R_3$, hence the triple (10) is correct. Therefore, the triple (1) is correct.

Therefore, the lemma is correct.

B. Proof of Lemma 3

Proof.

Step 1: prove that the initialization establishes the loop invariant, i.e., $\{P\}x := \texttt{false}; i := 0; v := 0\{\texttt{inv}\}.$

The proof of triple $\{P\}x := \texttt{false}; i := 0; v := 0\{\texttt{inv}\}$ is as follows.

$$\begin{split} &\operatorname{inv}[v:=0][i:=0][x:=\operatorname{false}] \\ &\equiv 0 \leq i \leq n \land (x = \operatorname{false} \lor (x = \operatorname{true} \land \\ & E[v] = e \land ((\operatorname{exe} = \operatorname{true} \land r > S[v]) \lor \\ & (\operatorname{exe} = \operatorname{false} \land r < S[v])))[v:=0][i:=0][x:=\operatorname{false}] \\ &\equiv 0 \leq i \leq n \land (x = \operatorname{false} \lor (x = \operatorname{true} \land \\ & E[0] = e \land ((\operatorname{exe} = \operatorname{true} \land r > S[0]) \lor \\ & (\operatorname{exe} = \operatorname{false} \land r < S[0]))))[i:=0][x:=\operatorname{false}] \\ &\equiv \operatorname{true} \land (x = \operatorname{false} \lor (x = \operatorname{true} \land \\ & E[0] = e \land ((\operatorname{exe} = \operatorname{true} \land r > S[0]) \lor \\ & (\operatorname{exe} = \operatorname{false} \land r < S[0])))[x:=\operatorname{false}] \\ &\equiv \operatorname{true} \land (\operatorname{true} \lor (\operatorname{false} \land \\ & E[0] = e \land ((\operatorname{exe} = \operatorname{true} \land r > S[0]) \lor \\ & (\operatorname{exe} = \operatorname{false} \land r < S[0])))][x:=\operatorname{false}] \\ &\equiv \operatorname{true} \land (\operatorname{true} \lor (\operatorname{false} \land \\ & E[0] = e \land ((\operatorname{exe} = \operatorname{true} \land r > S[0]) \lor \\ & (\operatorname{exe} = \operatorname{false} \land r < S[0])))) \\ &\equiv \operatorname{true} \end{split}$$

As $P \to \text{true}$, hence the triple $\{P\}x := \text{false}; i := 0; v := 0 \{\text{inv}\}$ is correct.

Step 2: prove that the loop body does not change the loop invariant, i.e., $\{inv \land i < n\}S\{inv\}$, where

$$S \equiv \text{ if } E[i] = e \land (\text{exe} = \text{true} \land r > S[i]) \lor \\ (\text{exe} = \text{false} \land r < S[i]) \text{ then} \\ v := i; \ x := \text{true}; \text{ fi}; \\ i := i + 1 \end{cases}$$

According to the composition rule [35], the proof of $\{inv \land i < n\}S\{inv\}$ is equivalent to prove the following two triples:

$$\{\operatorname{inv} \land i < n\}S_1\{R\} \tag{11}$$

$$\{R\}i := i + 1\{\texttt{inv}\}$$
(12)

where $R \equiv -1 \leq i \leq n-1 \land (x = \texttt{false} \lor (x = \texttt{true} \land E[v] = e \land ((\texttt{exe} = \texttt{true} \land r > S[v]) \lor (\texttt{exe} = \texttt{false} \land r < S[v])))),$

$$S_1 \equiv$$
 if B then $v := i; x :=$ true; fi

and $B \equiv E[i] = e \land ((exe = true \land r > S[i]) \lor (exe = false \land r < S[i])).$

$$\begin{split} & \operatorname{inv}[i:=i+1] \\ & \equiv 0 \leq i \leq n \land (x = \operatorname{false} \lor (x = \operatorname{true} \land \\ & E[v] = e \land ((\operatorname{exe} = \operatorname{true} \land r > S[v]) \lor \\ & (\operatorname{exe} = \operatorname{false} \land r < S[v])))[i:=i+1] \\ & \equiv -1 \leq i \leq n-1 \land (x = \operatorname{false} \lor (x = \operatorname{true} \land \\ & E[v] = e \land ((\operatorname{exe} = \operatorname{true} \land r > S[v]) \lor \\ & (\operatorname{exe} = \operatorname{false} \land r < S[v])))) \\ & \equiv R \end{split}$$

Hence, the triple (12) is correct.

The proof of triple (11) is equivalent to prove the following two triples:

$$\{\texttt{inv} \land i < n \land B\}v := i; x := \texttt{true}\{R\}$$
(13)

$$\{\texttt{inv} \land i < n \land \neg B\} \texttt{skip} \{R\}$$
(14)

The proof of triple (13) is as follows.

$$\begin{split} R[x := \texttt{true}][v := i] \\ &\equiv -1 \leq i \leq n - 1 \land (x = \texttt{false} \lor (x = \texttt{true} \land \\ E[v] = e \land ((\texttt{exe} = \texttt{true} \land r > S[v]) \lor \\ (\texttt{exe} = \texttt{false} \land r < S[v])))[x := \texttt{true}][v := i] \\ &\equiv -1 \leq i \leq n - 1 \land (E[v] = e \land ((\texttt{exe} = \texttt{true} \land r > S[v]) \\ \lor (\texttt{exe} = \texttt{false} \land r < S[v])))[v := i] \\ &\equiv -1 \leq i \leq n - 1 \land (E[i] = e \land ((\texttt{exe} = \texttt{true} \land r > S[i]) \\ \lor (\texttt{exe} = \texttt{false} \land r < S[i]))) \\ &\equiv R_1 \end{split}$$

$$\begin{split} & \operatorname{inv}\wedge i < n \wedge B \\ & \equiv 0 \leq i \leq n \wedge (x = \operatorname{false} \lor (x = \operatorname{true} \land \\ & E[v] = e \land ((\operatorname{exe} = \operatorname{true} \land r > S[v]) \lor \\ & (\operatorname{exe} = \operatorname{false} \land r < S[v]))) \land i < n \land \\ & E[i] = e \land (\operatorname{exe} = \operatorname{true} \land r > S[i]) \lor \\ & (\operatorname{exe} = \operatorname{false} \land r < S[i]) \\ & \equiv 0 \leq i < n \land (x = \operatorname{false} \lor (x = \operatorname{true} \land \\ & E[v] = e \land ((\operatorname{exe} = \operatorname{true} \land r > S[v]) \lor \\ & (\operatorname{exe} = \operatorname{false} \land r < S[v]))) \land \\ & E[i] = e \land (\operatorname{exe} = \operatorname{true} \land r > S[i]) \lor \\ & (\operatorname{exe} = \operatorname{false} \land r < S[v]))) \land \\ & E[i] = e \land (\operatorname{exe} = \operatorname{true} \land r > S[i]) \lor \\ & (\operatorname{exe} = \operatorname{false} \land r < S[i]) \end{split}$$

As *i* is an integer, hence $0 \le i < n \rightarrow -1 \le i \le n-1$. As inv $\land i < n \land B \rightarrow R_1$, hence the triple (13) is correct. Similarly, we prove the triple (14) as follows.

inv $\wedge i < n \wedge \neg B$

$$= 0 \le i \le n \land (x = \texttt{false} \lor (x = \texttt{true} \land E[v] = e \land ((\texttt{exe} = \texttt{true} \land r > S[v]) \lor (\texttt{exe} = \texttt{false} \land r < S[v]))) \land i < n \land \neg B$$

$$= 0 \le i < n \land (x = \texttt{false} \lor (x = \texttt{true} \land B))) \land i < n \land \neg B$$

$$= [v] = e \land ((\texttt{exe} = \texttt{true} \land r > S[v]) \lor (\texttt{exe} = \texttt{false} \land r < S[v]))) \land i < n \land \neg B$$

As $\operatorname{inv} \wedge i < n \wedge \neg B \rightarrow R$, hence the triple (14) is correct. Therefore, the triple (11) and triple $\{\operatorname{inv} \wedge i < n\}S\{\operatorname{inv}\}$ are correct.

Step 3: prove that the bound function decreases after each iteration, i.e., $\{inv \land i < n \land bd = z\}S\{bd < z\}$, where z is an integer.

According to the composition rule [35], the proof of $\{inv \land i < n \land bd = z\}S\{bd < z\}$ is equivalent to prove the following two triples:

$$\{\texttt{inv} \land i < n \land \texttt{bd} = z\}S_1\{n - i < z + 1\}$$
(15)

$$\{n - i < z + 1\}i := i + 1\{bd < z\}$$
(16)

The proof of triple (16) is as follows.

$$\begin{aligned} & \operatorname{bd} < z[i := i+1] \\ & \equiv n-i < z[i := i+1] \\ & \equiv n-i < z+1 \end{aligned}$$

Hence, the triple (16) is correct.

The proof of triple (15) is equivalent to prove the following two triples:

$$\{\texttt{inv} \land i < n \land \texttt{bd} = z \land B\}v := i; x := \texttt{true}\{n - i < z + 1\}$$
(17)

$$\{\texttt{inv} \land i < n \land \texttt{bd} = z \land \neg B\} \texttt{skip} \{n - i < z + 1\}$$
(18)

The proof of triple (17) is as follows.

$$\begin{aligned} n-i &< z+1[x:=\texttt{true}][v:=i] \\ &\equiv n-i < z+1 \\ &\texttt{inv} \wedge i < n \wedge \texttt{bd} = z \wedge B \end{aligned}$$

$$\equiv n-i = z \wedge \texttt{inv} \wedge i < n \wedge B$$

As $inv \wedge i < n \wedge bd = z \wedge B \rightarrow n - i < z + 1$, hence the triple (17) is correct.

Similarly, as $inv \wedge i < n \wedge bd = z \wedge \neg B \rightarrow n - i < z + 1$, hence the triple (18) is correct. Therefore, the triple (15) and triple $\{inv \wedge i < n \wedge bd = z\}S\{bd < z\}$ are correct.

Step 4: prove that the loop invariant implies the bound function is non-negative, i.e., $inv \rightarrow bd \ge 0$. As $bd \ge 0 \equiv n \ge i$, hence $inv \rightarrow bd \ge 0$.

Step 5: prove that the postcondition holds when the while loop terminates, i.e., $inv \land \neg(i < n) \rightarrow Q$.

$$\begin{split} & \operatorname{inv} \wedge \neg (i < n) \\ & \equiv 0 \leq i \leq n \wedge (x = \operatorname{false} \lor (x = \operatorname{true} \land \\ & E[v] = e \ \land \ ((\operatorname{exe} = \operatorname{true} \land r > S[v]) \lor \lor \\ & (\operatorname{exe} = \operatorname{false} \land r < S[v])))) \ \land i \geq n \\ & \equiv i = n \wedge (x = \operatorname{false} \lor (x = \operatorname{true} \land E[v] = e \land \\ & ((\operatorname{exe} = \operatorname{true} \land r > S[v]) \lor (\operatorname{exe} = \operatorname{false} \land r < S[v])))) \\ & \rightarrow Q \end{split}$$

C. Proof of Lemma 5

Proof. The proof of triple (4) is as follows.

$$\begin{split} &Q[x := a] \\ &\equiv (t = \texttt{stNum} \land x = 1) \lor (t \neq \texttt{stNum} \land x = t + 1)[x := a] \\ &\equiv (t = \texttt{stNum} \land a = 1) \lor (t \neq \texttt{stNum} \land a = t + 1) \\ &\equiv R \end{split}$$

Hence, the triple (4) is correct.

The proof of triple (3) is equivalent to prove the following two triples:

$$\{P \land t = \mathtt{stNum}\}a := 1\{R\}$$
(19)

$$\{P \land t \neq \mathtt{stNum}\}a := t + 1\{R\}$$
(20)

The proof of triple (19) is as follows.

$$\begin{split} R[a:=1] \\ &\equiv (t = \texttt{stNum} \land a = 1) \lor (t \neq \texttt{stNum} \land a = t+1)[a:=1] \\ &\equiv t = \texttt{stNum} \lor (t \neq \texttt{stNum} \land t = 0) \\ &\equiv R_1 \end{split}$$

As $P \wedge t = \text{stNum} \rightarrow R_1$, hence the triple (19) is correct. The proof of triple (20) is as follows.

$$\begin{split} R[a &:= t + 1] \\ &\equiv (t = \texttt{stNum} \land a = 1) \lor (t \neq \texttt{stNum} \land a = t + 1)[a := t + 1] \\ &\equiv (t = \texttt{stNum} \land t = 0) \lor t \neq \texttt{stNum} \\ &\equiv R_2 \end{split}$$

As $P \wedge t \neq \text{stNum} \rightarrow R_2$, hence the triple (20) is correct. Therefore, the triple (3) is correct.

Therefore, the lemma is correct.

D. Proof of Lemma 6

Proof. The proof of triple (5) is as follows.

$$\begin{split} &Q[x := \texttt{true}] \\ &\equiv (x = \texttt{true} \land O[t-1] = \texttt{st}) \lor \\ &(x = \texttt{false} \land O[t-1] \neq \texttt{st})[x := \texttt{true}] \\ &\equiv (\texttt{true} = \texttt{true} \land O[t-1] = \texttt{st}) \lor \\ &(\texttt{true} = \texttt{false} \land O[t-1] \neq \texttt{st}) \\ &\equiv O[t-1] = \texttt{st} \end{split}$$

As $P \wedge O[t-1] = st \rightarrow O[t-1] = st$, hence the triple (5) is correct.

Similarly, the proof of triple (6) is as follows.

$$\begin{split} &Q[x:=\texttt{false}]\\ &\equiv (x=\texttt{true} \land O[t-1]=\texttt{st}) \lor \\ &(x=\texttt{false} \land O[t-1]\neq\texttt{st})[x:=\texttt{false}]\\ &\equiv (\texttt{false}=\texttt{true} \land O[t-1]=\texttt{st}) \lor \\ &(\texttt{false}=\texttt{false} \land O[t-1]\neq\texttt{st})\\ &\equiv O[t-1]\neq\texttt{st} \end{split}$$

As $P \wedge O[t-1] \neq st \rightarrow O[t-1] \neq st$, hence the triple (6) is correct.

Therefore, the lemma is correct.

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