Rules and Axioms

Rule A1: Decomposition

$$\frac{\begin{array}{l}\vdash_p \{p\}S\{q\},\\ \vdash_t \{p\}S\{\texttt{true}\}\end{array}}{\{p\}S\{q\}}$$

Axiom A2: Invariance

$$\{p\}S\{p\}$$

where $free(p) \cap change(S) = \phi$

Rule A3: Disjunction

$$\frac{\{p\}S\{q\}, \{r\}S\{q\}}{\{p \vee r\}S\{q\}}$$

Rule A4: Conjunction

$$\frac{\{p_1\}S\{q_1\}, \{p_2\}S\{q_2\}}{\{p_1 \wedge p_2\}S\{q_1 \wedge q_2\}}$$

Rule A5: $\exists$-Introduction

$$\frac{\{p\}S\{q\}}{\{\exists x : p\}S\{q\}}$$

where x does not occur in S or in $free(q)$.

Rule A6: Invariance

$$\frac{\{r\}S\{q\}}{\{p \wedge r\}S\{p \wedge q\}}$$

where $free(p) \cap change(S) = \phi$

Rule A7: Substitution

$$\frac{\{p\}S\{q\}}{\{p[\overline{u} := \overline{t}]\}S\{q[\overline{u} := \overline{t}]\}}$$

where $\overline{u} \notin var(S) \wedge \overline{t} \notin change(S)$.

Proof System Rules

Axiom 1: Skip

$$\{p\}\texttt{skip}\ \{p\}$$

Axiom 2: Assignment

$$\{p[u := t]\}u := t\{p\}$$

Rule 3: Composition

$$\frac{\{p\}S_1\{r\}, \{r\}S_2\{q\}}{\{p\}S_1; S_2\{q\}}$$

Rule 4: Conditional

$$\frac{\{p \wedge B\}S_1\{q\}, \{p \wedge \neg B\}S_2\{q\}}{\{p\}\texttt{if}\ \ B\ \texttt{then}\ \ S_1\ \texttt{else}\ \ S_2\ \texttt{fi}\ \{q\}}$$

Rule 5: Loop

$$\frac{\{p \wedge B\}S\{p\}}{\{p\}\texttt{while}\ \ B\ \texttt{do}\ \ S\ \texttt{od}\ \{p \wedge \neg B\}}$$

Rule 6: Consequence

$$\frac{p \rightarrow p_1, \{p_1\}S\{q_1\}, q_1 \rightarrow q}{\{p\}S\{q\}}$$

Rule 7: Loop II

$$\frac{\begin{array}{l}\{p \wedge B\}S\{p\},\\ \{p \wedge b \wedge t = z\}S\{t < z\},\\ p \rightarrow t \geq 0\end{array}}{\{p\}\texttt{while}\ \ B\ \texttt{do}\ \ S\ \texttt{od}\ \ \{p \wedge \neg B\}}$$

where t is an integer expression and z is an integre variable that does not appear in p, B, t, or S.

Rule 8: Recursion

$$\frac{\begin{array}{l}\{p_1\}P_1\{q_1\},\ldots,\{p_n\}P_n\{q_n\} \vdash \{p\}S\{q\},\\ \{p_1\}P_1\{q_1\},\ldots,\{p_n\}P_n\{q_n\} \vdash \{p_i\}S_i\{q_i\}, i \in \{1,\ldots,n\},\end{array}}{\{p\}S\{q\}}$$

where $D = P_1 :: S_1, \ldots, P_n :: S_n$ (this rule is not covered in class).

Rule 9: Recursion II

(not covered in class)

Rule 10: Block

$$\frac{\{p\}\overline{x} := \overline{t}; S\{q\}}{\{p\}\texttt{begin local } \overline{x} := \overline{t}; S \texttt{ end } \{q\}}$$

where $var(\overline{x}) \cap free(q) = \phi$

Rule 11: Instantiation

(not covered in class)

Rule 12: Recursion III

(not covered in class)

Rule 13: Recursion IV

(not covered in class)

Axiom 14: Assignment to Instance Variables

$$\{p[u := t]\}u := t\{p\}$$

where u is a (simple or subscripted) instance variable.

Rule 15: Instantiation II — Method Instantiation

$$\frac{\{p\}y.m\{q\}}{\{p[y := s]\}s.m\{q[y := s]\}}$$

where D is the set of method declarations, $y \notin var(D)$ and $var(s) \cap change(D) = \phi$.

Rule 16: Recursion V

$$\frac{\begin{array}{l}\{p_1\}P_1\{q_1\},\ldots,\{p_n\}P_n\{q_n\} \vdash \{p\}S\{q\},\\ \{p_1\}P_1\{q_1\},\ldots,\{p_n\}P_n\{q_n\} \vdash \\ \qquad \{p_i\}\texttt{begin local this} := s_i; S_i\ \texttt{end}\ \{q_i\}, i \in \{1, ldots, n\}\end{array}}{\{p\}S\{q\}}$$

where $m_i :: S_i \in D$ for $i \in \{1, \ldots, n\}$. Its simplified version is

$$\frac{\{p\}\texttt{begin local this} := s; S\ \texttt{end}\ \{q\}}{\{p\}s.m\{q\}}$$

where $D = m :: S$.